The Higgs search of the MSSM with explicit CP violation at the LHC and ILC

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Abstract

We study the neutral Higgs sector of the minimal supersymmetric standard model (MSSM) with explicit CP violation at the one-loop level. We take into account the one-loop contributions by the top quark, the stop quarks, the bottom quark, the sbottom quarks, the tau lepton, the stau leptons, the W boson, the charged Higgs boson, the charginos, the Z boson, the neutral Higgs bosons, and the neutralinos. The production cross sections of the neutral Higgs boson are calculated to the leading order. The processes in our consideration are divided in two groups: the Higgs-strahlung and gluon fusion processes accessible at the CERN Large Hadron Collider (LHC), and the vector boson fusion and Higgs-strahlung processes accessible at the e^+e^- International Linear Collider (ILC). In particular, we investigate the dependence of these processes on the CP phase arising from the $U(1)$ factor of the gaugino mass in the neutralino mass matrix. We show that the cross sections of these processes vary by the range of $3\% - 19$ % as the CP phase changes from zero to π.

I. Introduction

The violation of CP symmetry has been observed in the neutral kaon system more than four decades ago [1]. In the Standard Model (SM), the CP violation can be induced by a complex phase in the Cabibbo-Kobayashi-Maskawa matrix for the charged weak current [2]. Alternatively, it is known that if a model possesses at least two Higgs doublets, the CP violation may occur in its Higgs sector through the mixing between the CP even and the CP odd states [3]. It is one of the characteristics of the minimal supersymmetric standard model (MSSM) that it should possess two Higgs doublets in order to give masses to the up-quark sector and the down-quark sector separately [4-7]. Thus, in principle, the MSSM may accommodate the CP violation by means of complex phases in its neutral Higgs sector. In practice, it has been found that the CP violation is impossible to occur either explicitly or spontaneously in the Higgs sector of the MSSM at the tree level, because the complex phases can always be eliminated by rotating the Higgs fields.

The possibility of CP violation in the MSSM at the one-loop level has been studied by many authors [8-38]. It has been noticed that the spontaneous CP violation in the Higgs sector of the MSSM at the one-loop level is disfavored because it requires a very light neutral Higgs boson, which has already been ruled out by experiments. The explicit CP violation, on the other hand, is viable by virtue of the radiative corrections due to the loops of relevant particles such as the quarks, the squarks, the W boson, the charged Higgs boson, the charginos, the Z boson, the neutral Higgs bosons, and the neutralinos. The radiative corrections by these particles yield the mixing between the CP even and the CP odd neutral Higgs bosons. Thus, it is possible to achieve the explicit CP violation in the radiatively corrected Higgs sector of the MSSM.

Recently, a number of studies have been devoted to the prospects for discovering neutral Higgs bosons in a general two Higgs doublet model (THDM) [39-41] and in the MSSM [34-36] in high-energy e^+e^- collisions within the context of explicit CP violation. In the MSSM with explicit CP violation, where the CP mixing among the CP even and the CP odd states is maximized, the cross sections for the neutral Higgs boson productions in e^+e^- collisions at $\sqrt{s} = 500$ and 800 GeV have been calculated [36]. Also, in the context of the explicit CP violation scenario the production cross sections of the neutral Higgs boson in hadron colliders have been calculated by considering the Higgs-strahlung process which is associated with the weak gauge bosons [37] and the gluon fusion process [38].

In this article, we investigate the phenomenological implication of the CP phase arising from the neutralino contribution which contributes to the CP mixing among the scalar and pseudoscalar Higgs bosons on the Higgs search at the future high-energy colliders, such as the CERN Large Hadron Collider (LHC) and the e^+e^- International Linear Collider (ILC). Within the framework of the explicit CP violation in the MSSM at the one-loop level, we calculate the production cross sections of three neutral Higgs bosons at the leading order in high-energy e^+e^- and PP collisions. At the LHC, the three dominant Higgs productions are considered to be the gluon fusion process and the associated Higgs-strahlung processes with the weak gauge bosons(Z, W). At the ILC, the three dominant mechanisms for the neutral Higgs productions are considered to be the vector boson (Z, W) fusion processes and the Higgs-strahlung process. We pay attention to the dependence

of the Higgs production on the CP phase whose appearance comes from the neutralino contribution to the tree-level Higgs sector. In the MSSM with explicit CP violation, the production cross sections of the neutral Higgs bosons at both LHC and ILC are shown to vary about 3%$-$19 % with respect to the variation of the CP phase, arising from the $U(1)$ factor of the gaugino mass parameter, from zero to π.

II. Higgs sector

The Higgs sector of the MSSM consists of two Higgs doublets, H_1 and H_2. In terms of these Higgs doublets, the superpotential of the MSSM is given by

$$W = h_b Q b^c_R H_1 + h_t Q t^c_R H_2 + h_\tau L H_1 \tau^c_R - \mu H_1 H_2 \ , \tag{1}$$

where we take into account only the third generation: Q and L are the SU(2) doublet quark and lepton superfields of the third generation respectively, t^c_R, b^c_R and τ^c_R are the SU(2) singlet top, bottom, and tau superfields respectively, h_t, h_b, h_τ are the Yukawa coupling coefficients of top, bottom, and tau superfields respectively, and μ is the Higgs mixing parameter with mass dimension.

The Higgs potential at the tree level reads

$$\begin{aligned} V^0 &= \frac{g_2^2}{8}(H_1^\dagger \vec{\sigma} H_1 + H_2^\dagger \vec{\sigma} H_2)^2 + \frac{g_1^2}{8}(|H_2|^2 - |H_1|^2)^2 \\ &\quad + m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - m_3^2 (H_1^T \epsilon H_2 + \text{H.c.}) \ , \end{aligned} \tag{2}$$

where ϵ is an antisymmetric 2×2 matrix with $\epsilon_{12} = 1$, $\vec{\sigma}$ denotes the three Pauli matrices, g_1 and g_2 are the U(1) and SU(2) gauge coupling constants respectively, and m_i^2 ($i = 1, 2, 3$) are the soft SUSY breaking masses, which may be assumed to be real without loss of generality. We choose m_3^2 to be positive. Using minimum conditions with respect to the neutral Higgs fields, we may eliminate m_1^2 and m_2^2. Thus, only m_3^2 remains as a free parameter.

The two Higgs doublets may be expressed as

$$\begin{aligned} H_1 &= \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 + h_1 + i h_3 \sin\beta \\ C^{+*} \sin\beta \end{pmatrix} , \\ H_2 &= \frac{1}{\sqrt{2}} \begin{pmatrix} C^+ \cos\beta \\ v_2 + h_2 + i h_3 \cos\beta \end{pmatrix} e^{i\phi_0} , \end{aligned} \tag{3}$$

where v_1 and v_2 are the vacuum expectation values (VEVs) of the neutral Higgs fields, $\tan\beta = v_2/v_1$, and ϕ_0 is the relative phase between the two Higgs doublets. The five physical Higgs fields are three real neutral Higgs fields h_1, h_2, h_3 and one complex charged Higgs field C^+ carrying two real degrees of freedom.

The tree-level masses of the fermions of the third generation are given as $m_t = h_t v \sin\beta/\sqrt{2}$, $m_b = h_b v \cos\beta/\sqrt{2}$, $m_\tau = h_\tau v \cos\beta/\sqrt{2}$, and the tree-level masses of the weak gauge boson are given as $m_W^2 = g_2^2 v^2/4$ and $m_Z^2 = (g_1^2 + g_2^2) v^2/4$, where $v = \sqrt{v_1^2 + v_2^2} = 246$ GeV. In terms of these masses, we can express the masses of the Higgs bosons and those of superpartners.

At the tree level, ϕ_0 may be taken to be zero. Thus, at the tree level, CP violation can be avoided in the MSSM and the three neutral Higgs bosons are divided into the CP even states h^0, H^0, and the CP odd state A^0. The tree-level mass of A^0 is

$$m_A^2 = \frac{2m_3^2 \cos\phi_0}{\sin 2\beta} \ ,$$

while the tree-level masses of h^0 and H^0 are

$$m_h^2, m_H^2 = \frac{1}{2}\left[m_Z^2 + m_A^2 \mp \sqrt{(m_Z^2 + m_A^2)^2 - 4m_Z^2 m_A^2 \cos^2 2\beta}\right] \ ,$$

where $m_h^2 \leq m_H^2$ is understood. The tree-level mass of the charged Higgs boson C^+ is

$$m_C^2 = m_W^2 + m_A^2 \ .$$

For the masses of superpartners, the tree-level masses of the scalar fermions of the third generation are

$$\begin{aligned}
m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2 &= m_t^2 + \frac{1}{2}(m_Q^2 + m_T^2) + \frac{m_Z^2}{4}\cos 2\beta \mp \sqrt{X_{\tilde{t}}} \ , \\
m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2 &= m_b^2 + \frac{1}{2}(m_Q^2 + m_B^2) - \frac{m_Z^2}{4}\cos 2\beta \mp \sqrt{X_{\tilde{b}}} \ , \\
m_{\tilde{\tau}_1}^2, m_{\tilde{\tau}_2}^2 &= m_\tau^2 + \frac{1}{2}(m_L^2 + m_E^2) - \frac{m_Z^2}{4}\cos 2\beta \mp \sqrt{X_{\tilde{\tau}}} \ , \qquad (4)
\end{aligned}$$

with

$$\begin{aligned}
X_{\tilde{t}} &= \left\{\frac{1}{2}(m_Q^2 - m_T^2) + \left(\frac{2}{3}m_W^2 - \frac{5}{12}m_Z^2\right)\cos 2\beta\right\}^2 \\
&\quad + m_t^2(A_t^2 + \mu^2\cot^2\beta - 2A_t\mu\cot\beta\cos\phi_t) \ , \\
X_{\tilde{b}} &= \left\{\frac{1}{2}(m_Q^2 - m_B^2) + \left(\frac{1}{12}m_Z^2 - \frac{1}{3}m_W^2\right)\cos 2\beta\right\}^2 \\
&\quad + m_b^2(A_b^2 + \mu^2\tan^2\beta - 2A_b\mu\tan\beta\cos\phi_b) \ , \\
X_{\tilde{\tau}} &= \left\{\frac{1}{2}(m_L^2 - m_E^2) + \left(\frac{3}{8}m_Z^2 - \frac{1}{2}m_W^2\right)\cos 2\beta\right\}^2 \\
&\quad + m_\tau^2(A_\tau^2 + \mu^2\tan^2\beta - 2A_\tau\mu\tan\beta\cos\phi_\tau) \ , \qquad (5)
\end{aligned}$$

where A_t, A_b, and A_τ are trilinear soft SUSY breaking parameters coming from the scalar quark and lepton sectors of the third generation, and m_T, m_B, and m_Q are the quark singlets and doublet soft masses, and m_E and m_L are the lepton singlet and doublet soft masses. The tree-level masses of the charginos are

$$m_{\tilde{\chi}1}^2 \ , m_{\tilde{\chi}2}^2 = \frac{1}{2}(M_2^2 + \mu^2) + m_W^2 \mp \sqrt{X_{\tilde{\chi}}} \ , \qquad (6)$$

with

$$\begin{aligned}
X_{\tilde{\chi}} &= \left\{\frac{1}{2}(M_2^2 - \mu^2) - m_W^2\cos 2\beta\right\}^2 \\
&\quad + 2m_W^2\cos^2\beta(M_2^2 + \mu^2\tan^2\beta + 2M_2\mu\tan\beta\cos\phi_c) \ . \qquad (7)
\end{aligned}$$

Note that the CP violating phases ϕ_t, ϕ_b, ϕ_τ, and ϕ_c, stemming from the stop quark, the sbottom quark, the stau lepton, and the chargino contributions, appear in the above expression. Finally, the tree-level masses of the neutralinos are given as the eigenvalues of the neutralino mass matrix

$$\mathcal{M}_{\tilde{\chi}^0} = \left(\begin{array}{cccc} M_1 e^{i\phi_1} & 0 & -g_1 v_1/2 & g_1 v_2/2 \\ 0 & M_2 & g_2 v_1/2 & -g_2 v_2/2 \\ -g_1 v_1/2 & g_2 v_1/2 & 0 & -\mu e^{i\phi_2} \\ g_1 v_2/2 & -g_2 v_2/2 & -\mu e^{i\phi_2} & 0 \end{array} \right) .$$

Here, too, two CP violating phases ϕ_1 and ϕ_2 are present. The CP phase ϕ_1 is the relative phase between M_1 and M_2, and ϕ_2 is the relative phase between M_2 and μ. Thus, ϕ_2 is identical to ϕ_c and the additional CP phase arising from the neutralino sector is only ϕ_1.

At the one-loop level, the explicit CP violation would occur and the neutral Higgs bosons can no longer have definite CP eigenvalues. The CP even states and the CP odd state are mixed to yield h_i (i = 1,2,3). We employ the effective potential method in order to calculate radiative corrections [42]. The one-loop effective potential is given as

$$V^1 = \sum_k \frac{c_k}{64\pi^2} (-1)^{2J_k} (2J_k + 1) \mathcal{M}_k^4(h_i) \left[\log \left(\frac{\mathcal{M}_k^2(h_i)}{\Lambda^2} \right) - \frac{3}{2} \right] ,$$

where the summation over k is carried out for all contributions, namely, the contributions of the top quark, the stop quarks, bottom quark, the sbottom quarks, the tau lepton, the stau leptons, the W boson, the charged Higgs boson, the charginos, the Z boson, the neutral Higgs bosons, and the neutralinos; and J_k is the spin of the corresponding particle, Λ is the renormalization scale in the modified minimal subtraction ($\overline{\rm MS}$) scheme, and c_k is the color factor $c_{\rm colour}$ multiplied by the charge factor $c_{\rm charge}$. The color factors for colored and uncolored particles are 3 and 1 respectively, and the charge factors for charged and neutral particles are 2 and 1 respectively. Note that in this one-loop effective potential $\mathcal{M}_k^2$ depends on the neutral Higgs fields h_i (i = 1, 2, 3), and it contains the tree-level masses of relevant particles and superpartners.

There is a CP-odd tadpole minimum condition with respect to h_3 at the one-loop level in the MSSM, which is given as

$$\begin{aligned} 0 \;\; = \;\; & m_3^2 \sin\phi_0 + \frac{3 m_t^2 \mu A_t \sin\phi_t}{16\pi^2 v^2 \sin^2\beta} f_1(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2) + \frac{3 m_b^2 \mu A_b \sin\phi_b}{16\pi^2 v^2 \cos^2\beta} f_1(m_{\tilde{b}_1}^2,\ m_{\tilde{b}_2}^2) \\ & + \frac{m_\tau^2 \mu A_\tau \sin\phi_\tau}{16\pi^2 v^2 \cos^2\beta} f_1(m_{\tilde{\tau}_1}^2,\ m_{\tilde{\tau}_2}^2) - \frac{m_W^2 \mu M_2 \sin\phi_c}{4\pi^2 v^2} f_1(m_{\tilde{\chi}_1}^2,\ m_{\tilde{\chi}_2}^2) \\ & + \sum_{k=1}^4 \frac{m_{\tilde{\chi}_k^0}^2}{4\pi^2 v^2} \left(\log \frac{m_{\tilde{\chi}_k^0}^2}{\Lambda^2} - 1 \right) \frac{E(m_{\tilde{\chi}_k^0}^2 - \mu^2)\mu}{\prod_{a\neq k} (m_{\tilde{\chi}_k^0}^2 - m_{\tilde{\chi}_a^0}^2)} , \end{aligned} \tag{8}$$

where the six terms on the right-hand side, in the order of the appearance, come from the tree-level Higgs potential, the one-loop contributions of the stop quark, the sbottom

quark, the stau lepton, the chargino, and the neutralinos, and

$$E = -\left[(m^2_{\tilde{\chi}^0_k} - M_1^2)M_2 m_W^2 \sin\phi_2 + (m^2_{\tilde{\chi}^0_k} - M_2^2)M_1(m_Z^2 - m_W^2)\sin(\phi_1+\phi_2)\right] \ ,$$

and the dimensionless function $f_1(m_x^2, m_y^2)$ is defined as

$$f_1(m_x^2,\ m_y^2) = \frac{2}{(m_x^2 - m_y^2)}\left\{m_x^2 \log\frac{m_x^2}{\Lambda^2} - m_y^2 \log\frac{m_y^2}{\Lambda^2}\right\} - 2 \ .$$

At the one-loop level, the matrix elements of M_{ij} in the (h_1, h_2, h_3) basis are given by

$$M_h = \left(\begin{array}{ccc} M_{11} & M_{12} & M_{13} \\ M_{12} & M_{22} & M_{23} \\ M_{13} & M_{23} & M_{33} \end{array}\right) \ , \tag{9}$$

where

$$\begin{aligned}
M_{11} &= m_Z^2 \cos^2\beta + \bar{m}_A^2 \sin^2\beta + M_{11}^1 \ , \\
M_{22} &= m_Z^2 \sin^2\beta + \bar{m}_A^2 \cos^2\beta + M_{22}^1 \ , \\
M_{33} &= \bar{m}_A^2 + M_{33}^1 \ , \\
M_{12} &= -(m_Z^2 + \bar{m}_A^2)\cos\beta\sin\beta + M_{12}^1 \ , \\
M_{13} &= M_{13}^1 \ , \\
M_{23} &= M_{23}^1 \ .
\end{aligned} \tag{10}$$

Here the mass parameter $\bar{m}_A$ is defined as

$$\begin{aligned}
\bar{m}_A^2 &= \frac{2}{\sin 2\beta}\Bigg[m_3^2\cos(\phi_0) + \frac{3m_t^2\mu A_t\cos\phi_t}{16\pi^2 v^2\sin^2\beta} f_1(m^2_{\tilde{t}_1},\ m^2_{\tilde{t}_2}) \\
&\quad + \frac{3m_b^2\mu A_b\cos\phi_b}{16\pi^2 v^2\cos^2\beta} f_1(m^2_{\tilde{b}_1},\ m^2_{\tilde{b}_2}) + \frac{m_\tau^2\mu A_\tau\cos\phi_\tau}{16\pi^2 v^2\cos^2\beta} f_1(m^2_{\tilde{\tau}_1},\ m^2_{\tilde{\tau}_2}) \\
&\quad + \frac{m_W^2\mu M_2\cos\phi_c}{4\pi^2 v^2} f_1(m^2_{\tilde{\chi}_1},\ m^2_{\tilde{\chi}_2}) \\
&\quad + \sum_{k=1}^{4}\frac{m^2_{\tilde{\chi}^0_k}}{4\pi^2 v^2}\left\{\log\left(\frac{m^2_{\tilde{\chi}^0_k}}{\Lambda^2}\right) - 1\right\}\frac{E_h}{\prod\limits_{a\neq k}(m^2_{\tilde{\chi}^0_k} - m^2_{\tilde{\chi}^0_a})}\Bigg] \ ,
\end{aligned} \tag{11}$$

with

$$\begin{aligned}
E_h &= (m^2_{\tilde{\chi}^0_k} - M_1^2)(m^2_{\tilde{\chi}^0_k} - \mu^2)M_2\mu m_W^2\cos\phi_2 \\
&\quad + (m^2_{\tilde{\chi}^0_k} - M_2^2)(m^2_{\tilde{\chi}^0_k} - \mu^2)M_1\mu(m_Z^2 - m_W^2)\cos(\phi_1+\phi_2) \ .
\end{aligned} \tag{12}$$

In M_h, the matrix elements $M_{i3} = M_{i3}^1$ $(i = 1, 2)$ represent the mixing between the scalar and the pseudoscalar components. Thus, nonvanishing of M_{i3} indicates that the mixing occurs at the one-loop level. There is no mixing at the tree level. Explicit expressions of M_{ij}^1 are given in Ref. [27].

The squared mass of the lightest neutral Higgs boson, $m^2_{h_1}$, is given as the smallest eigenvalue of the one-loop squared-mass matrix for the neutral Higgs bosons. The upper

bound on $m_{h_1}^2$ is obtained by noticing that the smallest eigenvalue of a positive symmetric matrix cannot exceed the smaller eigenvalue of its upper left 2×2 submatrix [43]. Thus, the upper bound on $m_{h_1}^2$ is given as

$$m_{h_1}^2 \leq m_{h_1,\ \max}^2 = m_Z^2 \cos^2 2\beta + \delta m_{\tilde{t}}^2 + \delta m_{\tilde{b}}^2 + \delta m_{\tilde{\tau}}^2 + \delta m_{\tilde{\chi}}^2 + \delta m_h^2 + \delta m_{\tilde{\chi}^0}^2 \ , \tag{13}$$

where the first term comes from the tree-level Higgs potential while the other terms come from the one-loop corrections due to the top quark, the stop quarks, the bottom quark, the sbottom quarks, tau lepton, the stau leptons, the W boson, the charged Higgs boson, the charginos, the Z boson, the neutral Higgs bosons, and the neutralinos. Explicitly, they are given as follows:

$$\begin{aligned}
\delta m_{\tilde{t}}^2 \ = \ & - \frac{3\Delta_{\tilde{t}}^2}{16\pi^2 v^2} \frac{f_2(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2)}{(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)^2} + \frac{3m_t^4}{4\pi^2 v^2} \log\left(\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4}\right) \\
& + \frac{3m_Z^2 \cos 2\beta}{64\pi^2 v^2} (\cos 2\beta m_Z^2 + 8m_t^2) \log\left(\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{\Lambda^4}\right) \\
& + \frac{3\cos^2 2\beta}{16\pi^2 v^2} \left(\frac{4m_W^2}{3} - \frac{5m_Z^2}{6}\right)^2 f_1(m_{\tilde{t}_1}^2,\ m_{\tilde{t}_2}^2) \\
& + \frac{3\Delta_{\tilde{t}}}{16\pi^2 v^2} (4m_t^2 + \cos 2\beta m_Z^2) \frac{\log(m_{\tilde{t}_2}^2/m_{\tilde{t}_1}^2)}{(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)} \ , \\
\delta m_{\tilde{b}}^2 \ = \ & - \frac{3\Delta_{\tilde{b}}^2}{16\pi^2 v^2} \frac{f_2(m_{\tilde{b}_1}^2,\ m_{\tilde{b}_2}^2)}{(m_{\tilde{b}_2}^2 - m_{\tilde{b}_1}^2)^2} + \frac{3m_b^4}{4\pi^2 v^2} \log\left(\frac{m_{\tilde{b}_1}^2 m_{\tilde{b}_2}^2}{m_b^4}\right) \\
& + \frac{3m_Z^2 \cos 2\beta}{64\pi^2 v^2} (\cos 2\beta m_Z^2 - 8m_b^2) \log\left(\frac{m_{\tilde{b}_1}^2 m_{\tilde{b}_2}^2}{\Lambda^4}\right) \\
& + \frac{3\cos^2 2\beta}{16\pi^2 v^2} \left(\frac{m_Z^2}{6} - \frac{2m_W^2}{3}\right)^2 f_1(m_{\tilde{b}_1}^2,\ m_{\tilde{b}_2}^2) \\
& + \frac{3\Delta_{\tilde{b}}}{16\pi^2 v^2} (4m_b^2 - \cos 2\beta m_Z^2) \frac{\log(m_{\tilde{b}_2}^2/m_{\tilde{b}_1}^2)}{(m_{\tilde{b}_2}^2 - m_{\tilde{b}_1}^2)} \ , \\
\delta m_{\tilde{\tau}}^2 \ = \ & - \frac{\Delta_{\tilde{\tau}}^2}{16\pi^2 v^2} \frac{f_2(m_{\tilde{\tau}_1}^2,\ m_{\tilde{\tau}_2}^2)}{(m_{\tilde{\tau}_2}^2 - m_{\tilde{\tau}_1}^2)^2} + \frac{m_\tau^4}{4\pi^2 v^2} \log\left(\frac{m_{\tilde{\tau}_1}^2 m_{\tilde{\tau}_2}^2}{m_\tau^4}\right) \\
& + \frac{m_Z^2 \cos 2\beta}{64\pi^2 v^2} (\cos 2\beta m_Z^2 - 8m_\tau^2) \log\left(\frac{m_{\tilde{\tau}_1}^2 m_{\tilde{\tau}_2}^2}{\Lambda^4}\right) \\
& + \frac{\cos^2 2\beta}{16\pi^2 v^2} \left(\frac{3m_Z^2}{4} - m_W^2\right)^2 f_1(m_{\tilde{\tau}_1}^2,\ m_{\tilde{\tau}_2}^2) \\
& + \frac{\Delta_{\tilde{\tau}}}{16\pi^2 v^2} (4m_\tau^2 - \cos 2\beta m_Z^2) \frac{\log(m_{\tilde{\tau}_2}^2/m_{\tilde{\tau}_1}^2)}{(m_{\tilde{\tau}_2}^2 - m_{\tilde{\tau}_1}^2)} \ , \\
\delta m_{\tilde{\chi}}^2 \ = \ & \frac{\Delta_{\tilde{\chi}}^2}{8\pi^2 v^2} \frac{f_2(m_{\tilde{\chi}_1}^2,\ m_{\tilde{\chi}_2}^2)}{(m_{\tilde{\chi}_2}^2 - m_{\tilde{\chi}_1}^2)^2} + \frac{m_W^4}{4\pi^2 v^2} \log\left(\frac{m_W^6 m_C^2}{m_{\tilde{\chi}_1}^4 m_{\tilde{\chi}_2}^4}\right) \\
& - \frac{\cos^2 2\beta m_W^4}{2\pi^2 v^2} f_1(m_{\tilde{\chi}_1}^2,\ m_{\tilde{\chi}_2}^2) - \frac{\Delta_{\tilde{\chi}} m_W^2}{2\pi^2 v^2} \frac{\log(m_{\tilde{\chi}_2}^2/m_{\tilde{\chi}_1}^2)}{(m_{\tilde{\chi}_2}^2 - m_{\tilde{\chi}_1}^2)} \ ,
\end{aligned}$$

$$
\begin{aligned}
\delta m_h^2 &= -\frac{v^2\Delta_h}{32\pi^2}\frac{f_2(m_h^2,\ m_H^2)}{(m_H^2-m_h^2)^2}+\frac{m_Z^2}{64\pi^2v^2}(m_A^2\sin^2 2\beta+8m_Z^2\cos^2\beta\cos 2\beta)f_1(m_h^2,\ m_H^2)\\
&\quad+\frac{m_Z^2\Delta_h}{16\pi^2}\frac{\log(m_H^2/m_h^2)}{(m_H^2-m_h^2)}+\frac{m_Z^4}{32\pi^2v^2}\log\left(\frac{m_h^2m_H^2}{\Lambda^4}\right)\ ,\\
&\quad+\frac{3m_Z^4}{8\pi^2v^2}\log\left(\frac{m_Z^2}{\Lambda^2}\right)-\frac{m_Z^4\cos^2 2\beta}{128\pi^2v^2}(\cos 4\beta-5)\log\left(\frac{m_A^2}{\Lambda^2}\right)\ ,\\
\delta m_{\tilde\chi^0}^2 &= \sum_{k=1}^4\frac{m_{\tilde\chi_k^0}^2}{16\pi^2}\left(\log\frac{m_{\tilde\chi_k^0}^2}{\Lambda^2}-1\right)\frac{B_2m_{\tilde\chi_k^0}^4+C_2m_{\tilde\chi_k^0}^2+D_2}{v^2\prod\limits_{a\neq k}^4(m_{\tilde\chi_k^0}^2-m_{\tilde\chi_a^0}^2)}\\
&\quad-\sum_{k=1}^4\frac{1}{16\pi^2}\log\left(\frac{m_{\tilde\chi_k^0}^2}{\Lambda^2}\right)\frac{(Am_{\tilde\chi_k^0}^6+Bm_{\tilde\chi_k^0}^4+Cm_{\tilde\chi_k^0}^2+D)^2}{v^2\left[\prod\limits_{a\neq k}^4(m_{\tilde\chi_k^0}^2-m_{\tilde\chi_a^0}^2)\right]^2}\\
&\quad+\sum_{k=1}^4\frac{m_{\tilde\chi_k^0}^2}{8\pi^2}\left(\log\frac{m_{\tilde\chi_k^0}^2}{\Lambda^2}-1\right)(Am_{\tilde\chi_k^0}^6+Bm_{\tilde\chi_k^0}^4+Cm_{\tilde\chi_k^0}^2+D)\\
&\quad\times\left[\frac{1}{v^2}\sum_{a\neq k}^4\frac{(Am_{\tilde\chi_a^0}^6+Bm_{\tilde\chi_a^0}^4+Cm_{\tilde\chi_a^0}^2+D)}{(m_{\tilde\chi_k^0}^2-m_{\tilde\chi_a^0}^2)^3\prod\limits_{c\neq k\neq a}^4(m_{\tilde\chi_k^0}^2-m_{\tilde\chi_c^0}^2)(m_{\tilde\chi_a^0}^2-m_{\tilde\chi_c^0}^2)}\right]\ ,
\end{aligned}
\tag{14}
$$

where

$$
\begin{aligned}
\Delta_{\tilde t} &= 2m_t^2(A_t^2-2A_t\mu\cot\beta\cos\phi_t+\mu^2\cot^2\beta)\\
&\quad+\cos 2\beta\left(\frac{4m_W^2}{3}-\frac{5m_Z^2}{6}\right)\left\{m_Q^2-m_T^2+\cos 2\beta\left(\frac{4m_W^2}{3}-\frac{5m_Z^2}{6}\right)\right\}\ ,\\
\Delta_{\tilde b} &= 2m_b^2(A_b^2-2A_b\mu\tan\beta\cos\phi_b+\mu^2\tan^2\beta)\\
&\quad+\cos 2\beta\left(\frac{m_Z^2}{6}-\frac{2m_W^2}{3}\right)\left\{m_Q^2-m_B^2+\cos 2\beta\left(\frac{m_Z^2}{6}-\frac{2m_W^2}{3}\right)\right\}\ ,\\
\Delta_{\tilde\tau} &= 2m_\tau^2(A_\tau^2-2A_\tau\mu\tan\beta\cos\phi_\tau+\mu^2\tan^2\beta)\\
&\quad+\cos 2\beta\left(\frac{3m_Z^2}{4}-m_W^2\right)\left\{m_L^2-m_E^2+\cos 2\beta\left(\frac{3m_Z^2}{4}-m_W^2\right)\right\}\ ,\\
\Delta_{\tilde\chi} &= 2m_W^2(M_2^2+\mu^2+2M_2\mu\sin 2\beta\cos\phi_c+2\cos^2 2\beta m_W^2)\ ,\\
\Delta_h &= \frac{m_Z^2}{2v^2}\{(3+\cos 4\beta)m_Z^2-(1+3\cos 4\beta)m_A^2\}\ ,
\end{aligned}
$$

and

$$
\begin{aligned}
A &= -4m_Z^2\ ,\\
B &= 4M_1^2m_W^2+4M_2^2(m_Z^2-m_W^2)+4m_Z^2(m_Z^2+\mu^2)\\
&\quad-4\mu\sin\beta\sin 2\beta\left[M_2m_W^2\cos\phi_2-M_1(m_W^2-m_Z^2)\cos(\phi_1+\phi_2)\right]\ ,\\
C &= -4m_Z^4\mu^2\sin^2 2\beta-4M_1^2m_W^2(m_W^2+\mu^2)+4M_2^2(m_Z^2-m_W^2)(m_W^2-m_Z^2-\mu^2)\\
&\quad+8M_1M_2m_W^2(m_W^2-m_Z^2)\cos\phi_1+4M_2m_W^2\mu(M_1^2+\mu^2)\sin 2\beta\cos\phi_2\\
&\quad+4M_1\mu(m_Z^2-m_W^2)(M_2^2+\mu^2)\sin 2\beta\cos(\phi_1+\phi_2)\ ,\\
D &= +4M_1^2m_W^4\mu^2\sin^2 2\beta+4M_2^2\mu^2(m_Z^2-m_W^2)^2\sin^2 2\beta\\
&\quad+8M_1M_2m_W^2\mu^2(m_Z^2-m_W^2)\sin^2 2\beta\cos\phi_1-4M_1^2M_2m_W^2\mu^3\sin 2\beta\cos\phi_2
\end{aligned}
$$

$$+ 4M_1M_2^2\mu^3(m_W^2 - m_Z^2)\sin 2\beta\cos(\phi_1+\phi_2) \ ,$$

and

$$\begin{aligned}
B_2 &= 8m_Z^4 \ , \\
C_2 &= -8M_1^2m_W^4 - 8M_2^2(m_Z^2-m_W^2)^2 + 16M_1M_2m_W^2(m_W^2-m_Z^2)\cos\phi_1 \ , \\
D_2 &= 8M_1^2m_W^4\mu^2\sin^2 2\beta + 8M_2^2\mu^2(m_Z^2-m_W^2)^2\sin^2 2\beta \\
&\quad + 16M_1M_2m_W^2\mu^2(m_Z^2-m_W^2)\sin^2 2\beta\cos\phi_1 \ .
\end{aligned}$$

Here the scale independent function $f_2(m_x^2, m_y^2)$ is defined by

$$f_2(m_x^2, m_y^2) = \frac{m_y^2+m_x^2}{m_y^2-m_x^2}\log\frac{m_y^2}{m_x^2} - 2 \, .$$

Note that the above upper bound on the lightest neutral Higgs boson mass does not depend on the tree-level mass of the pseudoscalar Higgs boson.

It is also possible to express analytic forms for the squared masses of the three Higgs bosons as

$$m_{h_n}^2 = \frac{1}{3}\mathrm{Tr}(M_h) + 2\sqrt{W}\cos\left\{\frac{\theta+2n\pi}{3}\right\} \ , \ (n=1,2,3) \tag{15}$$

with

$$\theta = \cos^{-1}\left(\frac{Y}{\sqrt{W^3}}\right) \ , \tag{16}$$

where

$$\begin{aligned}
W &= -\frac{1}{18}\{\mathrm{Tr}(M_h)\}^2 + \frac{1}{6}\mathrm{Tr}(M_h^2) \ , \\
Y &= -\frac{5}{108}\{\mathrm{Tr}(M_h)\}^3 + \frac{1}{12}\mathrm{Tr}(M_h)\mathrm{Tr}(M_h^2) + \frac{1}{2}\det(M_h) \ .
\end{aligned} \tag{17}$$

One can calculate the transformation matrix for the neutral Higgs bosons from the orthogonality condition. The elements of the transformation matrix are given by

$$U_{ij} = \frac{o_{ij}}{\sqrt{\sum_{k=1}^3 o_{ik}^2}} \ , \tag{18}$$

where

$$\begin{aligned}
o_{ii} &= 1 \ , \ (i=1,2,3), \\
o_{12} &= \frac{(m_{h_1}^2 - M_{11})M_{23} + M_{12}M_{13}}{(m_{h_1}^2 - M_{22})M_{13} + M_{12}M_{23}} \ , \\
o_{13} &= \frac{(m_{h_1}^2 - M_{11})(m_{h_1}^2 - M_{22}) - M_{12}^2}{(m_{h_1}^2 - M_{22})M_{13} + M_{12}M_{23}} \ , \\
o_{21} &= \frac{(m_{h_2}^2 - M_{22})M_{13} + M_{12}M_{23}}{(m_{h_2}^2 - M_{11})M_{23} + M_{12}M_{13}} \ ,
\end{aligned}$$

$$
\begin{aligned}
o_{23} &= \frac{(m_{h_2}^2 - M_{11})(m_{h_2}^2 - M_{22}) - M_{12}^2}{(m_{h_2}^2 - M_{22})M_{23} + M_{12}M_{13}} \ , \\
o_{31} &= \frac{(m_{h_3}^2 - M_{33})M_{12} + M_{23}M_{13}}{(m_{h_3}^2 - M_{11})M_{23} + M_{12}M_{13}} \ , \\
o_{32} &= \frac{(m_{h_3}^2 - M_{11})(m_{h_3}^2 - M_{33}) - M_{13}^2}{(m_{h_3}^2 - M_{11})M_{23} + M_{12}M_{13}} \ .
\end{aligned} \tag{19}
$$

III. Higgs productions

The most important channels for the productions of neutral Higgs bosons at the LHC are: the gluon fusion process via the triangular loop of top quark $PP \to gg \to h_i$, the Higgs-strahlung process mediated by W boson $PP \to q\bar{q}' \to Wh_i$, and the Higgs-strahlung process mediated by Z boson $PP \to q\bar{q} \to Zh_i$. We denote the production cross sections for these processes as $\sigma(h_i)$, $\sigma(Wh_i)$, and $\sigma(Zh_i)$ respectively. At the lowest order, the cross sections of these processes are related to the SM cross sections for the corresponding SM Higgs boson production channels as [44-47]

$$
\begin{aligned}
&\sigma(h_i) = K_i^2 \sigma_{\rm SM}(PP \to gg \to h_i) \ , \\
&\sigma(Wh_i) = R_i^2 \sigma_{\rm SM}(PP \to q\bar{q}' \to Wh_i) \ , \\
&\sigma(Zh_i) = R_i^2 \sigma_{\rm SM}(PP \to q\bar{q} \to Zh_i) \ ,
\end{aligned} \tag{20}
$$

where K_i and R_i (i = 1,2,3) are defined as

$$
\begin{aligned}
K_i &= \frac{U_{2i}}{\sin\beta} \ , \\
R_i &= \cos\beta U_{1i} + \sin\beta U_{2i} \ ,
\end{aligned} \tag{21}
$$

with U_{ij} being the elements of the diagonalization matrix for the 3×3 neutral Higgs boson mass matrix. Using the ortho-normality condition of U_{ij}, one can show that $\sum_{i=1}^{3} R_i^2 = 1$. The factor K_i comes from the coupling of the ith neutral Higgs boson to a top quark pair normalized by the corresponding SM coupling, and the factor R_i comes from the coupling of the ith neutral Higgs boson to a Z (W) boson pair normalized by the corresponding SM coupling.

For the numerical analysis, we take $\sin^2\theta_W = 0.23$ for weak-mixing angle, $G_F = 1.166 \times 10^{-5}$ for Fermi coupling constant, $m_Z = 91.187$ GeV for the Z boson mass, $m_W = 80.423$ GeV for the W boson mass, $m_t = 175$ GeV for the mass of the top quark, and $m_b = 4.5$ GeV for the mass of the bottom quark. The renormalization and factorization scales are taken to be the same as the neutral Higgs boson mass. The parton densities are set as CTEQ6M [48,49].

We would like to concentrate on a particular region of the parameter space of the MSSM with explicit CP violation. The relevant free parameters are Λ, $\tan\beta$, μ, m_Q, m_T, m_B, m_L, m_E, A_t, A_b, A_τ, M_1, M_2, ϕ_t, ϕ_b, ϕ_τ, $\phi_c(\phi_2)$, and ϕ_1. We set $m_Q = m_L$, $m_T = m_B = m_E$, $A_t = A_b = A_\tau$, and $\phi_t = \phi_b = \phi_\tau = \phi_c(\phi_2)$ for simplicity. At the electroweak scale one can take the relation between $U(1)$ and $SU(2)$ gaugino masses to be $M_1 = 5\tan^2\theta_W M_2/3$.

We are interested in examining the dependence of the contribution of the neutralinos on the production cross section of the neutral Higgs bosons. The contributions of the neutralino loops depend crucially on the CP phase ϕ_1. In other words, the CP phase ϕ_1 occurs only in the expressions from the neutralino contributions. We search for some parameter region where the three neutral Higgs bosons becomes relatively light with masses below 150 GeV by using a random number generating function. In such a region, the production cross sections of the neutral Higgs bosons mainly depend on the relevant coupling constants rather than the neutral Higgs boson masses. We calculate the neutral Higgs boson masses as ϕ_1 varies from zero to π, for $\tan\beta = 28.2$, $\bar{m}_A = 135$ GeV, $\mu = 458$ GeV, $m_Q = 544$ GeV, $m_T = 480$ GeV, $A_t = 932$ GeV, $M_2 = 390$ GeV, and $\phi_t = 2.58$. In this case, the neutral Higgs boson masses depend weakly on the CP phase ϕ_1 and are given approximately by $m_{h_1} \approx 131$ GeV, $m_{h_2} \approx 132$ GeV, and $m_{h_3} \approx 135$ GeV.

We calculate the value of K_i^2 and R_i^2 as functions of the CP phase ϕ_1, for the above parameter values. These values are plotted in Fig.1 as the phase ϕ_1 varies from zero to π. Note that the phase dependence of K_i^2 is carried by U_{2i} via mass matrix elements and that of R_i^2 is carried by both U_{1i} and U_{2_i}. The production cross sections of the neutral Higgs bosons via the gluon fusion process mediated by the triangular loop of the top quark are modified by K_i from the corresponding SM cross section, and the production cross sections of the neutral Higgs bosons via the Higgs-strahlung process mediated by the vector bosons are modified by R_i^2 from the corresponding SM cross section. One can find from Fig. 1 that the factors vary 12 % for K_1^2, 19 % for K_2^2, and 3.6 % for K_3^2, while the factors vary 13 % for R_1^2, 18 % for R_2^2, and 4.2 % for R_3^2.

It is well known that the gluon fusion process at the LHC is the most dominant mechanism for the production of the neutral Higgs boson in the SM. It is also well known that the Higgs-strahlung process mediated by the W boson at the LHC is more dominant than that mediated by the Z boson for the production of the neutral Higgs boson in the SM.

In Fig. 2, the production cross sections of th neutral Higgs bosons for the processes in Eq. (20) at the LHC are plotted as functions of the CP phase ϕ_1. The values of the other parameters are taken to be the same as in Fig. 1.

In this figure, the solid curves represent the production cross sections of the neutral Higgs bosons via the gluon fusion process which is mediated with the triangular loop of the top quark. One can see that $\sigma(h_1)$, $\sigma(h_2)$, and $\sigma(h_3)$ vary about 13.0 %, 19.5 %, and 3.8 % respectively as ϕ_1 goes from zero to π. The production cross section of the ith neutral Higgs boson via the gluon fusion depends significantly on the factors K_i.

The dashed curves represent the production cross sections of the neutral Higgs bosons via the Higgs-strahlung process mediated by the W boson. One can see that $\sigma(Wh_1)$, $\sigma(Wh_2)$, and $\sigma(Wh_3)$ vary about 14.1 %, 18.3 %, and 3.8 % respectively as ϕ_1 goes from zero to π. The dotted curves represent the production cross sections of the neutral Higgs bosons via the Higgs-strahlung process mediated by the Z boson. One can also see that $\sigma(Zh_1)$, $\sigma(Zh_2)$, and $\sigma(Zh_3)$ vary about 14.1 %, 12.7 %, and 3.4 % respectively as ϕ_1 goes from zero to π. The production cross sections of the ith neutral Higgs boson via the the Higgs-strahlung processes mediated by the vector bosons depend significantly on the factors R_i. Note that the variations of the above cross sections are different from those

of the factors K_i and R_i as can be seen by comparing Figs. 1 and 2. This is because the corresponding SM cross sections also depend on the phase ϕ_1 indirectly through the masses of the neutral Higgs bosons.

We now turn to the production processes of the Higgs boson accessible at the ILC. At the ILC, the dominant production mechanisms for the neutral Higgs boson are the Higgs-strahlung process and the vector boson (W, Z) fusion processes. At the lowest order, the cross sections of these processes are related to the SM cross sections for the corresponding SM Higgs boson production processes as [50-53]

$$\begin{aligned}
&\sigma(ZZh_i) = R_i^2 \sigma_{\rm SM}(e^+e^- \to Zh_i) \ , \\
&\sigma(\nu\nu h_i) = R_i^2 \sigma_{\rm SM}(e^+e^- \to \nu_e\bar{\nu}_e h_i) \ , \\
&\sigma(eeh_i) = R_i^2 \sigma_{\rm SM}(e^+e^- \to e^+e^- h_i) \ .
\end{aligned} \tag{22}$$

It is well known that at the center of mass energy of $\sqrt{s} \sim 200$ GeV, the Higgs-strahlung process is the most dominant mechanism for the production of the neutral Higgs boson in the SM. It is also well known that at the center of mass energy of $\sqrt{s} \sim 500$ GeV, the WW fusion process is the most dominant mechanism for the production of the neutral Higgs boson in the SM.

In Fig. 3, the production cross sections of the neutral Higgs bosons for the processes in Eq. (22) at the ILC with $\sqrt{s} = 500$ GeV (ILC500) are plotted as functions of the CP phase ϕ_1. The values of the other parameters are taken to be the same as in Fig. 1.

In this figure, the solid curves represent the production cross sections of the neutral Higgs bosons via the Higgs-strahlung process at the ILC500. One can see that $\sigma(ZZh_1)$, $\sigma(ZZh_2)$, and $\sigma(ZZh_3)$ vary about 14.1 %, 18.3 %, and 4 % respectively as ϕ_1 goes from zero to π.

The dashed curve represent the production cross sections of the neutral Higgs bosons via the WW fusion process. One can see that $\sigma(\nu\nu h_1)$, $\sigma(\nu\nu h_2)$, and $\sigma(\nu\nu h_3)$ vary about 14.1 %, 18.3 %, and 4 % respectively as ϕ_1 goes from zero to π.

The dotted curve represent the production cross sections of the neutral Higgs bosons via the ZZ fusion process. One can also see that $\sigma(eeh_1)$, $\sigma(eeh_2)$, and $\sigma(eeh_3)$ vary about 14.1 %, 16.7 %, and 3.8 % respectively as ϕ_1 goes from zero to π. The production cross sections of the ith neutral Higgs bosons via the three dominant processes at the ILC500 depend significantly on the factors R_i.

Here again, one can see that the variations of the above cross sections are different from those of the factor R_i as can be seen by comparing Figs. 1 and 3. It is interesting to notice that the variations for the solid curves are almost the same as those for the dashed curves, which indicates that the dependences of $\sigma_{\rm SM}(e^+e^- \to Zh_i)$ and $\sigma_{\rm SM}(e^+e^- \to \nu_e\bar{\nu}_e h_i)$ on the CP phase ϕ_1 are almost identical.

IV. Conclusions

We have studied the Higgs sector of the MSSM where the CP symmetry is explicitly violated at the one-loop level. We have calculated the mass matrix of the three neutral Higgs bosons and the analytic forms of their masses. In this calculation, we have taken

into account all the relevant one-loop contributions including those of the top quark, the stop quarks, the bottom quark, the sbottom quarks, the tau lepton, the stau leptons, the W boson, the charged Higgs boson, the charginos, the Z boson, the neutral Higgs bosons, and the neutralinos.

We have also studied the production processes of the neutral Higgs bosons. Three dominant channels accessible at the LHC for the neutral Higgs boson production are the Higgs-strahlung processes mediated by the Z boson, the W boson, and the gluon fusion process involving the triangular loop of the top quark, and three dominant channels accessible at the ILC for the neutral Higgs boson production are the Higgs-strahlung process, WW and ZZ fusion processes. We have calculated the cross sections of these processes. The range of the variations of the cross sections due to the variation of the CP phase ϕ_1 turns out to be as high as about 19.5 %. This indicates the nontrivial dependence of the Higgs production processes on the CP phase ϕ_1 which arises from the neutralino sector.

In the MSSM with explicit CP violation, radiative corrections of the neutralino loop to the tree-level Higgs sector give rise to an important contribution for the CP mixing between the scalar and pseudoscalar Higgs bosons. Therefore, we suggest that, as in the explicit CP violation scenario, such a CP mixing effect arising from the neutralino contribution might have important phenomenological implications for the Higgs search at both the LHC and the ILC.

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[1] J. H. Christen, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Lett. **13**, 138 (1964).

[2] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[3] S. Weinberg, Phys. Rev. Lett. **37**, 657 (1976).

[4] P. Fayet and S. Ferrara, Phys. Rep. **32**, 249 (1977); P. Fayet, Phys. Rep. **105**, 21 (1984).

[5] H. P. Nilles, Phys. Rep. **110**, 1 (1984).

[6] H. E. Haber and G. L. Kane, Phys. Rep. **117**, 75 (1985).

[7] J. F. Gunion, H. E. Haber, G. L. Kane, and S. Dawson, *The Higgs Hunters' Guide* (Addison-Wesley, Redwood City, CA, USA, 1990).

[8] N. Maekawa, Phys. Lett. B **282**, 387 (1992).

[9] A. Pomarol, Phys. Lett. B **287**, 331 (1992).

[10] N. Haba, Phys. Lett. B **398**, 305 (1997).

[11] O. Lebedev, Mod. Phys. Lett. A **13**, 735 (1998).

[12] O. Lebedev, Eur. Phys. J. C **4**, 363 (1998).

[13] M. Brhlik and G. L. Kane, Phys. Lett. B **437**, 331 (1998).

[14] A. Pilaftsis and C. E. M. Wagner, Nucl. Phys. B **553**, 3 (1999).

[15] D. A. Demir, Phys. Rev. D **60**, 095007 (1999).

[16] S. Y. Choi and J. S. Lee, Phys. Rev. D **61**, 015003 (1999).

[17] M. Carena, J. Ellis, A. Pilaftsis, and C. E. M. Wagner, Nucl. Phys. B **586**, 92 (2000).

[18] G. L. Kane and L. T. Wang, Phys. Lett. B **488**, 383 (2000).

[19] T. Ibrahim and P. Nath, Phys. Rev. D **63**, 035009 (2001); Phys. Rev. D **66**, 015005 (2002); Phys. Rev. D **68**, 015008 (2003); Phys. Rev. D **70**, 035006 (2004).

[20] S. Heinemeyer, Eur. Phys. J. C **22**, 521 (2001).

[21] M. Boz and N. K. Pak, Phys. Rev. D **65**, 075014 (2002).

[22] M. Carena, J. Ellis, A. Pilaftsis, and C. E. M. Wagner, Nucl. Phys. B **625**, 345 (2002).

[23] E. Christova, H. Eberl, S. Kraml, and W. Majerotto, Nucl. Phys. B **639**, 263 (2002).

[24] N. Ghodbane, S. Katsanevas, I. Laktineh, J. Rosiek, Nucl. Phys. B **647**, 190 (2002).

[25] M. Boz, J. Phys. G **28**, 2377 (2002).

[26] S. W. Ham, S. K. Oh, E. J. Yoo, and H. K. Lee, J. Phys. G **27**, 1 (2001).

[27] S. W. Ham, S. K. Oh, E. J. Yoo, C. M. Kim, and D. Son, Phys. Rev. D **68**, 055003 (2003).

[28] M. Carena and H. E. Haber, Prog. Part. Nucl. Phys. **50**, 63 (2003).

[29] A. Dedes and A. Pilaftsis, Phys. Rev. D **67**, 015012 (2003).

[30] M. Carena, J. R. Ellis, S. Mrenna, A. Pilaftsis, and C. E. M. Wagner, Nucl. Phys. B **659**, 145 (2003).

[31] J. S. Lee, A. Pilaftsis, M. Carena, S. Y. Choi, M. Drees, J. R. Ellis, and C. E. M. Wagner, Comput. Phys. Commun. **156**, 283 (2004).

[32] J.R. Ellis, J. S. Lee, and A. Pilaftsis, Phys. Rev.D **70**, 075010, (2004).

[33] M. Argyrou, A. B. Lahanas, and D. V. Nanopoulos, Phys. Rev. D **70**, 095008, (2004); Erratum-ibid.D **70**, 119902, (2004).

[34] A.G. Akeroyd and A. Arhrib, Phys. Rev. D **64** 095018 (2001).

[35] M. Carena, J. Ellis, A. Pilaftsis, and C. E. M. Wagner, Phys. Lett. B **495**, 155 (2000).

[36] A. Arhrib, Phys. Rev. D **67**, 015003 (2003).

[37] A. Arhrib, D.K. Ghosh, and Otto C.W. Kong, Phys. Lett. B **537**, 217 (2002).

[38] S.Y. Choi, K. Hagiwara, and J.S. Lee, Phys. Lett. B **529**, 212 (2002).

[39] A. Mendez and A. Pomarol, Phys. Lett. B **272**, 313 (1991).

[40] J. F. Gunion, B. Grzadkowski, H. E. Haber, and J. Kalinowski, Phys. Rev. Lett. **79**, 982 (1997).

[41] B. Grzadkowski, J. F. Gunion, and J. Kalinowski, Phys. Rev. D **60**, 075001 (2000); B. Grzadkowski, J. F. Gunion, and J. Kalinowski, Phys. Lett. B **480**, 287 (2000).

[42] S. Coleman and E. Weinberg, Phys. Rev. D **7**, 1888 (1973).

[43] M. Drees, Int. J. Mod. Phys. A **4**, 3635 (1989).

[44] H.M. Georgi, S.L. Glashow, M.E. Machacek, D.V. Nanopoulos, Phys. Rev. Lett. **40**, 692 (1978).

[45] M. Spira, A. Djouadi, D. Graudenz, P.M. Zerwas, Nucl. Phys. B **453**, 17 (1995).

[46] M. Spira, Fortsch. Phys. **46**, 203 (1998).

[47] R.V. Harlander, Phys. Lett. B **492**, 74 (2000).

[48] H.L. Lai, J. Botts, J. Huston, J.G. Morfin, J.F. Owens, J.W. Qiu, W.K. Tung, H. Weerts, Phys. Rev. D **51**, 4763 (1995).

[49] J. Pumplin, D.R. Stump, J. Huston, H.L. Lai, P. Nadolsky, and W.K. Tung, JHEP 0207, 012 (2002); D. Stump, J. Huston, J. Pumplin, W.K. Tung, H.L. Lai, S. Kuhlmann, and J.F. Owens, JHEP 0310, 046 (2003).

[50] E. Accomando *et. al.*, Phys. Rep. **299**, 1 (1998).

[51] A. Djouadi, PM/98-17, GDR-S-012 (1998).

[52] T. Abe *et al.*, American Linear Collider Working Group, hep-ex/0106056.

[53] TESLA, Part III: Physics at an e^+e^- Linear Collider, edited by R. D. Heuer, D. Miller, F. Richard, and P. M. Zerwas, hep-ph/0106315.

FIGURE CAPTION

FIG. 1. : The factors K_i^2 and R_i^2 ($i = 1, 2, 3$) as functions of ϕ_1, for $\tan\beta = 28.2$, $\bar{m}_A = 135$ GeV, $\mu = 458$ GeV, $m_Q = m_L = 544$ GeV, $m_T = 480$ GeV, $A_t = 932$ GeV, $M_2 = 390$ GeV, and $\phi_t = 2.58$.

FIG. 2. : The cross sections for h_i ($i = 1, 2, 3$) productions as functions of ϕ_1, via the gluon fusion process mediated by the triangular loop of the top quark (solid curves), via the Higgs-strahlung processes mediated by the W boson (dashed curves) and the Z boson (dotted curves). The other parameters are the same as in Fig. 1.

FIG. 3. : The cross sections for h_i ($i = 1, 2, 3$) productions as functions of ϕ_1, via the Higgs-strahlung process (solid curves), the WW fusion process (dashed curves), and the WW fusion process (dotted curves) at the ILC with $\sqrt{s} = 500$ GeV. The other parameters are the same as in Fig. 1.

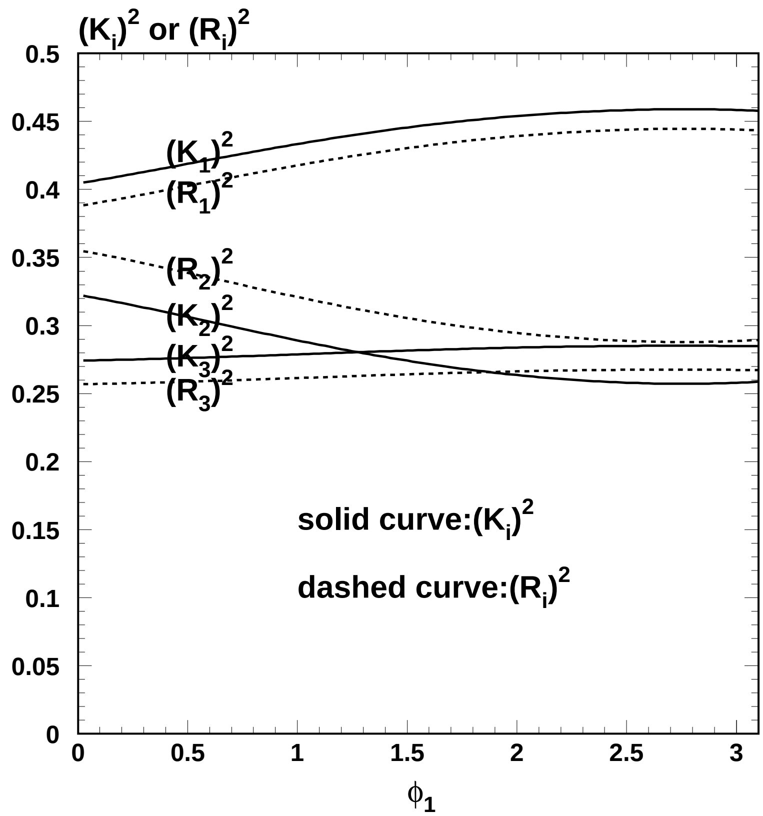

FIG. 1: The factors K_i^2 and R_i^2 $(i = 1, 2, 3)$ as functions of ϕ_1, for $\tan\beta = 28.2$, $\bar{m}_A = 135$ GeV, $\mu = 458$ GeV, $m_Q = m_L = 544$ GeV, $m_T = 480$ GeV, $A_t = 932$ GeV, $M_2 = 390$ GeV, and $\phi_t = 2.58$.

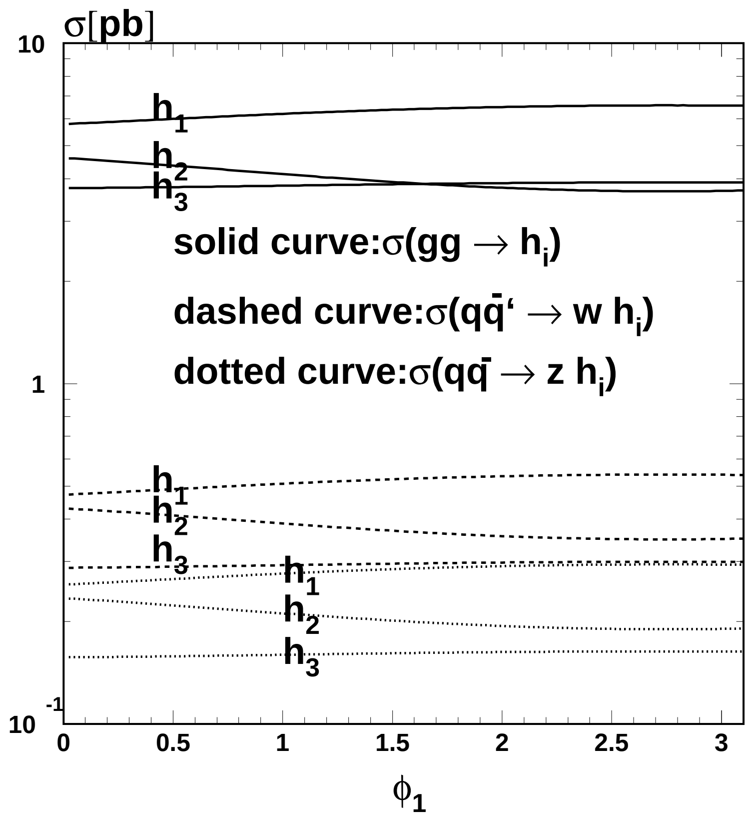

FIG. 2: The cross sections for h_i ($i = 1, 2, 3$) productions as functions of ϕ_1, via the gluon fusion process mediated by the triangular loop of the top quark (solid curves), via the Higgs-strahlung processes mediated by the W boson (dashed curves) and the Z boson (dotted curves). The other parameters are the same as in Fig. 1.

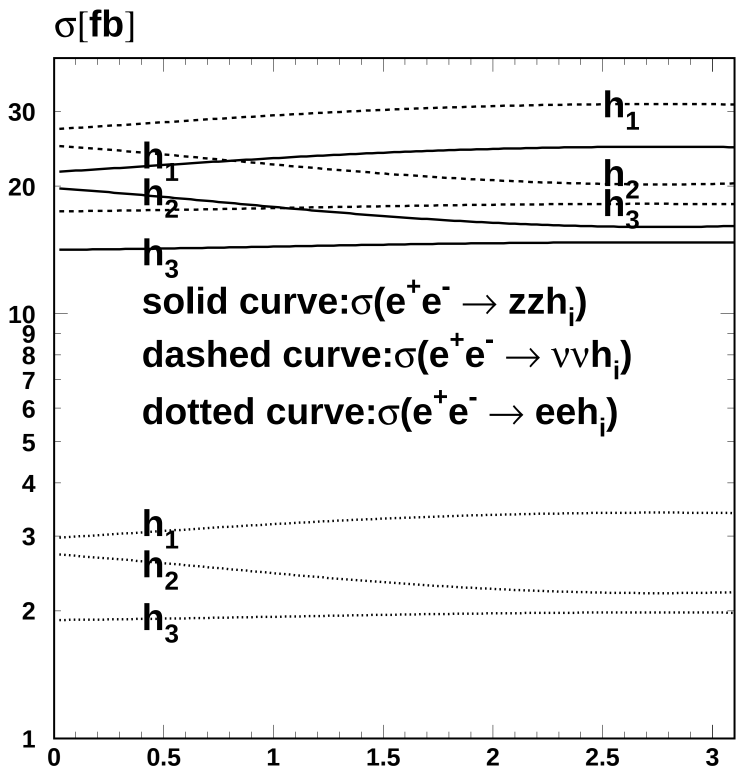

FIG. 3: The cross sections for h_i ($i = 1, 2, 3$) productions as functions of ϕ_1, via the Higgs-strahlung process (solid curves), the WW fusion process (dashed curves), and the WW fusion process (dotted curves) at the ILC with $\sqrt{s} = 500$ GeV. The other parameters are the same as in Fig. 1.